Jonathan Crabtree **How al-Khwarizmi multiplied fractions** 1st English Translation

<12> Capitulum aliud in multiplicatone fractionum et divisione earum. *Text courtesy Menso Folkerts* [1]	<12> Another chapter on the multiplication of fractions and their division. *Translation courtesy Peter Crabtree*
⟨12.3⟩ Cum ergo volueris multiplicare tres et dimidium in VIII et tribus partibus de XI, scribe tres.	<12.3> So if you want to multiply three and {a one} half by eight and three {eleventh parts}, write 'three'.
Postea pone sub eis unum et sub uno duo, et cum hoc feceris, iam posuisti tres et dimidium, quia dimidium est una pars ex duabus, quemadmodum unum minutum est sexagesima unius.	Then put under it (lit. them) a 'one', and under the 'one' 'two', and when you have done this you have written down 'three and {a one} half', since a half is one part out of two, just as a minute is a sixtieth of one.
Post hec scribe in alia parte VIII et sub eis III et sub tribus XI, et cum hoc feceris, iam posuisti VIII et tres partes de XI. 3 8 1 3 2 11	Next write on the other side 'eight' and under that (lit. them) 'three' and under the 'three' 'eleven', and when you have done that you have written down 'eight and three elevenths'. 3 8 1 3 2 11
Et sic reddas unumquemque ex eis de genere ultime differentie, hoc est, multiplicabis tria in duobus, ad que refertur unum, augesque eis unum, et sic erunt medietates.	And thus rewrite each of them according to the type in the bottom position, that is, you multiply the 'three' by the 'two' (to which the 'one' refers) and then add one: these will be halves.
Erunt igitur VII medietates, quas scribes in loco trium et destrues tria atque unum quod est sub eis.	So there are seven halves which you write in place of the 'three' and cross out the 'three' and the 'one' which is beneath it.
Multiplicabis VIII quoque in undecim, ad que referentur tria, et adde super ea tria, et sic reddes ea partes de XI.	Also you multiply the 'eight' by the 'eleven' (to which the 'three' refers) and then add three and in this way you make them elevenths.
Eruntque LX⟨L⟩I, que scribes in loco VIII et destrues VIII atque tria que sunt sub eis.	There are ninety-one [elevenths] which you write in place of the 'eight', crossing out the 'eight' and the 'three'.
Deinde multiplicabis genus medietatum, que sunt duo, in genere partium, que sunt XI, et erunt X⟨X⟩II.	Next you multiply the type of 'half', which is 2, by the type of 'eleventh', which is 11, to get 22.
Eritque hoc ex genere secundorum, et servabis illud.	This is the type of the parts (of unity) which is retained.
Item multiplicabis VII medietates in LX⟨L⟩ una parte XI, et erunt DCXXXVII.	Next you multiply 7 (halves) by 91 (elevenths) getting 637.
Et sunt etiam ex genere secundorum, que divides per XXII, et erunt eiusdem generis.	And these are the type of parts of unity which are 'twentysecondths', and they are (all) of the same type.
Divide igitur unum ex eis super alium, et quod exierit erit numerus integer, et quod remanserit erunt partes unius de illo numero quem dividis.	Thus divide one of them by the other and what results will be a whole number, with a remainder of parts of unity of the type of the divising number.
Et hoc est quod exivit tibi de multiplicatione, XXVIII scilicet et XX una pars ex XX duabus partibus unius.	And this is what you got out of the multiplication, 28 and 21 twentysecondth parts of unity.
Et similiter erit universa multiplicatio fractionum.	And all multiplication of fractions goes similarly.

STEP-BY-STEP INTERPRETATION

To multiply 3 and 1/2 by 8 and 3/11, write down the following:

3	8
1	3
2	11

From the depiction of 3 and 1/2 in the left column, we calculate the number of ‘halves’ as (3 x 2) + 1 = 7
From the depiction of 8 and 3/11 in the right column, we calculate the number of ‘elevenths’ as (8 x 11) + 3 = 91
The 3 is thus covered in ‘dust’ and replaced by 7 and the 8 is covered in dust and replaced by 91.
The 1 and 3 are presumably both erased and covered in dust as they now serve no purpose.

7	91
2	11

Next, multiply the two numbers in the bottom line, giving 22, which is the ‘type’ of the parts of unity we now have, namely, ‘twentysecondths’. Then, multiply the two numbers in the top line, getting 637. This is the number of ‘22ndths’ we have.

637

22

Next, 22 into 637 goes 28 wholes (units, ones) with 21 remaining, which are ‘22ndths’.
So, from the method of al-Khwarizmi, the result of 3 and 1/2 multiplied by 8 and 3/11 is 28 and 21/22.

We compare the method of multiplication of mixed fractions in the oldest extant Arabic book on Hindu arithmetic, ‘Kitab al-Fusul fi al-Hisab al-Hindi’, written circa 952 CE, by Abu'l Hasan Ahmad ibn Ibrahim Al-Uqlidisi. Al-Uqlidisi’s Arabic is available in English, courtesy of A. S. Saidan [2] from a 12th Century manuscript. In this, we read:

Multiplying a Number with Fractions by a Number with Fractions

We say that the way is to draw them, make the fractions of one number each, combine numbers to fractions, multiply and divide by the product of the two numbers of which the parts are derived.

For example, we want to multiply 7 and a half by 5 and a third. We assume them like this:

7	5
1	1
2	3

. We multiply 7 by two and add the one, which is the one-half; that becomes 15 with 2 below. We multiply 5 by 3 and add the third, which is one; it becomes 16 with 3 below. We multiply 16 by 15 and divide by 6. The outcome is 40.

Al-Uqlidisi’s approach follows al-Kwarizmi’s. With modern symbols and the distributive law, we get
(3 + 1/2) x (8 + 3/11)
= (3 x 8) + (3 x 3/11) + (1/2 x 8) + (1/2 x 3/11)
= 24 + 9/11 + 4 + 3/22
= 28 + 18/22 + 3/22
= 28 and 21/22

[1] *Die älteste lateinische Schrift über das indische Rechnen nach al-Ḫwārizmī*, (Latin and German) Menso Folkerts; Paul Kunitzsch; Hispanic Society of America, München : Verlag der Bayerischen Akademie der Wissenschaften, 1997.
[2] *The Arithmetic of Al-uqlidisi The Story of Hindu-arabic Arithmetic As Told in Kitab Al-fusul Fi Al-hisab Al-hindi*. A. S. Saidan, Springer Verlag 2013.
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